IJESRT: 7(10), October, 2018 ISSN: 2277-9655

International Journal of Engineering Sciences & Research Technology

(A Peer Reviewed Online Journal)
Impact Factor: 5.164

IJESRT

Website: www.ijesrt.com Mail: editor@ijesrt.com

Correspondence Address: 116, Sukhdev Nagar Ext-1, Airport Road, Indore – 452005, Madhya Pradesh, INDIA

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

MULTIPLICATIVE DAKSHAYANI INDICES

V. R. Kulli
Department of Mathematics, Gulbarga University, Gulbarga 585 106, India

DOI: 10.5281/zenodo.1471580

ABSTRACT

We introduce the multiplicative Dakshayani indices, multiplicative plus Dakshayani indices and general multiplicative Dakshayani index of a graph. We initiate a study of these new invariants.

Keywords: Dakshayani indices, multiplicative Dakshayani indices, networks.
Mathematics Subject Classification : 05*C*05, 05*C*07, 05*C*12.

1. INTRODUCTION

Let G be a simple connected graph with n vertices and m edges with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The complement $\overline{G}$ of G is the graph with vertex set $V(G)$ in which two vertices are adjacent if they are not adjacent in G. Clearly $d_{\overline{G}}(u) = n - 1 - d_G(u)$ For additional definitions and notations, the readers are referred to [1]. A topological index is a numerical parameter mathematically derived from the graph structure. In [2], Narumi-Katayama conceived a simple based multiplicative structure

$$NK(G) = \prod_{u \in V(G)} d_G(u),$$

which is referred to as the Narumi-Katayama index. This index was studied in [3, 4, 5].

In [6, 7], Todeschini et al. have proposed multiplicative variant of the first Zagreb index, which is defined as follows

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2.$$

This index was studied in [8, 9].

The multiplicative F-index of a graph G is defined as

$$FII(G) = \prod_{u \in V(G)} d_G(u)^3.$$

In [10], Kulli introduced the first and second Dakshayani indices, defined as

$$DK_1(G) = \sum_{u \in V(G)} d_{\overline{G}}(u) \frac{1}{d_G(u)}.$$

$$DK_2(G) = \sum_{u \in V(G)} d_{\overline{G}}(u) \frac{1}{d_G(u)^2}.$$

The general Dakshayani index is introduced by Kulli in [10], defined as

$$DK^a(G) = \sum_{u \in V(G)} d_{\overline{G}}(u) d_G(u)^a \tag{1}$$

where a is a real number.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

We now introduce the first and second multiplicative Dakshayani indices, defined as

$$DK_1II(G)=\prod_{u\in V(G)} d_{\overline{G}}(u)\frac{1}{d_G(u)}, \qquad DK_2II(G)=\prod_{u\in V(G)} d_{\overline{G}}(u)\frac{1}{d_G(u)^2}.$$

Also we propose the first and second multiplicative plus Dakshayani indices, defined as

$$DK_1^+II(G)=\prod_{u\in V(G)} d_{\overline{G}}(u)d_G(u), \qquad DK_2^+II(G)=\prod_{u\in V(G)} d_{\overline{G}}(u)d_G(u)^2.$$

We now propose the general multipliative Dakshayani index of a graph G, defined as

$$DK^aII(G)=\prod_{u\in V(G)} d_{\overline{G}}(u)d_G(u)^a. \qquad (2)$$

In this paper, we compute the Dakshayani indices, multiplicative Dakshayani indices of oxide, honeycomb, silicate, chain silicate networks. For more information about networks see [11].

2. OXIDE NETWORKS

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. An oxide network of dimension 5 is presented in Figure 1.

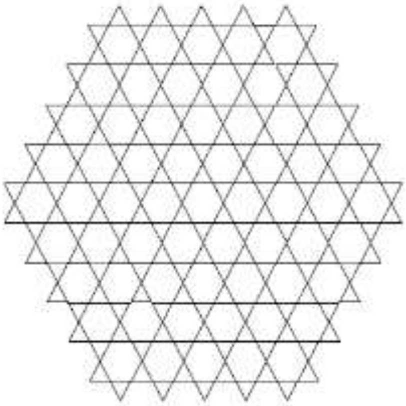

Figure 1. Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. By calculation, we obtain that G has $9n^2 + 3n$ vertices. In G there are two types of vertices based on the degree of vertices as given in Table 1.

Table 1. Vertex partitions of G and $\overline{G}$.

$d_G(u) \setminus u \in V(G)$	2	4
$d_{\overline{G}}(u) \setminus u \in V(\overline{G})$	$9n^2 + 3n - 3$	$9n^2 + 3n - 5$
Number of vertices	$6n$	$6n^2 - 3n$

Theorem 1. The general Dakshayani index of an oxide network OX_n is given by

$$DK^a(OX_n) = 2^a \times 6n(9n^2+3n-3) + 4^a(9n^2-3n)(9n^2+3n-5) \qquad (3)$$

Proof: Let G be the graph of an oxide network OX_n. By using equation (1) and Table 1, we deduce

$$DK^{a}(OX_n)=\sum_{u\in V(G)} d_{\bar{G}}(u)d_G(u)^{a}$$

$$=2^{a}(9n^2+3n-3)6n+4^{a}(9n^2+3n-5)(9n^2-3n).$$

We obtain the following results by using Theorem 1.

Corollary 1.1. The first Dakshayani index of OX_n is given by

$$DK_1(OX_n)=\frac{81}{4}n^4+27n^3-\frac{9}{2}n^2+\frac{15}{4}n-9.$$

Proof: Put $a = -1$ in equation (3), we get the desired result.

Corollary 1.2. The second Dakshayani index of OX_n is given by

$$DK_2(OX_n)=\frac{81}{16}n^4+\frac{27}{2}n^3+\frac{9}{4}n^2+\frac{15}{16}n-\frac{9}{2}.$$

Proof: Put $a = -2$ in equation (3), we get the desired result.

In the following theorem, we determine the general multiplicative Dakshayani index of OX_n.

Theorem 2. The general multiplicative Dakshayani index of an oxide network OX_n is given by

$$DK^{a}II(OX_n)=2^{18an^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}. \tag{4}$$

Proof: Let G be the graph of an oxide network OX_n. By using equation (2) and Table 1, we derive

$$DK^{a}II(OX_n)=\prod_{u\in V(G)} d_{\bar{G}}(u)d_G(u)^{a}$$

$$=\left[2^{a}(9n^2+3n-3)\right]^{6n}\times\left[4^{a}(9n^2+3n-5)\right]^{9n^2-3n}.$$

$$=2^{18an^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}.$$

The following results are obtained by using Theorem 2.

Corollary 2.1. The first multiplicative Dakshayani index of OX_n is given by

$$DK_1II(OX_n)=\left(\frac{1}{2}\right)^{18n^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}.$$

Proof: Put $a = -1$ in equation (4), we get the desired result.

Corollary 2.2. The second multiplicative Dakshayani index of OX_n is given by

$$DK_2II(OX_n)=\left(\frac{1}{4}\right)^{18n^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}.$$

Proof: Put $a = -2$ in equation (4), we get the desired result.

Corollary 2.3. The first multiplicative plus Dakshayani index of OX_n is given by

$$DK_1^{+}II(OX_n)=2^{18n^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}.$$

Proof: Put $a = 1$ in equation (4), we get the desired result.

Corollary 2.4. The second multiplicative plus Dakshayani index of OX_n is given by

$$DK_2^{+}II(OX_n)=2^{36n^2}\times(9n^2+3n-3)^{6n}\times(9n^2+3n-5)^{9n^2-3n}.$$

Proof: Put $a = 2$ in equation (4), we get the desired result.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

3. HONEYCOMB NETWORKS

Honeycomb networks are useful in Chemistry and Computer Graphics. A honeycomb network of dimension n is symbolized by HC_n, where n is the number of hexagons between central and boundary hexagons. A 4-dimensional honeycomb network is presented in Figure 2.

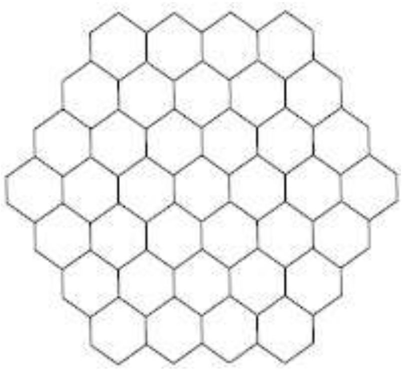

Figure 2. A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices. In G, there are two types of vertices based on the degree of vertices as given in Table 2.

Table 2. Vertex partition of G and $\overline{G}$

$d_G(u) \setminus u \in V(G)$	2	3
$d_{\overline{G}}(u) \setminus u \in V(\overline{G})$	$6n^2 - 3$	$6n^2 - 4$
Number of vertices	$6n$	$6n^2 - 6n$

In the following theorem, we compute the general Dakshayani index of HC_n.

Theorem 3. The general Dakshayani index of a honeycomb network HC_n is given by

$$DK^a(HC_n) = 2^a(6n^2 - 3)6n + 3^a(6n^2 - 4)(6n^2 - 6n). \quad (5)$$

Proof: Let G be the graph of a honeycomb network HC_n. By using equation (1) and Table 2, we deduce

$$DK^a(HC_n) = \sum_{u \in V(G)} \left(d_{\overline{G}}(u) d_G(u)\right)^a$$

$$= 2^a(6n^2 - 3)6n + 3^a(6n^2 - 4)(6n^2 - 6n).$$

We obtain the following results by using Theorem 3.

Corollary 3.1. The first Dakshayani index of HC_n is given by

$$DK_1(HC_n) = 12n^4 + 6n^3 - 8n^2 - n.$$

Proof: Put $a = -1$ in equation (5), we get the desired result.

Corollary 3.2. The second Dakshayani index of HC_n is given by

$$DK_2(HC_n) = 4n^4 + 5n^3 - \frac{9}{3}n^2 - \frac{11}{6}n.$$

Proof: Put $a = -2$ in equation (5), we get the desired result.

In the following theorem, we compute the general multiplicative Dakshayani index of HC_n.

Theorem 4. The general multiplicative Dakshayani index of a honeycomb network HC_n is given by

$$DK^a II(HC_n) = 2^{6an} \times 3^{a(6n^2-6n)} \times (6n^2-3)^{6n} \times (6n^2-4)^{6n^2-6n}. \quad (6)$$

Proof: Let G be the graph of a honeycomb network HC_n. By using equation (2) and Table 2, we derive

$$DK^a II(HC_n) = \prod_{u \in V(G)} d_{\bar{G}}(u) d_G(u)^a$$

$$= \left[2^a(6n^2-3)\right]^{6n} \times \left[3^a(6n^2-4)\right]^{6n^2-6n}.$$

$$= 2^{6an} \times 3^{a(9n^2-3n)} \times (6n^2-3)^{6n} \times (6n^2-4)^{6n^2-6n}.$$

Corollary 4.1. The first multiplicative Dakshayani index of HC_n is given by

$$DK_1 II(HC_n) = 2^{6n^2-12n} \times 3^{-(6n^2-12n)} \times (2n^2-1)^{6n} \times (3n^2-2)^{6n^2-6n}.$$

Proof: Put $a = -1$ in equation (6), we get the desired result.

Corollary 4.2. The second multiplicative Dakshayani index of HC_n is given by

$$DK_2 II(HC_n) = 2^{6n^2-18n} \times 3^{-(12n^2-18n)} \times (2n^2-1)^{6n} \times (3n^2-2)^{6n^2-6n}.$$

Proof: Put $a = -2$ in equation (6), we get the desired result.

Corollary 4.3. The first multiplicative plus Dakshayani index of HC_n is given by

$$DK_1^+ II(HC_n) = 2^{6n^2} \times 3^{6n^2} \times (2n^2-1)^{6n} \times (3n^2-2)^{6n^2-6n}.$$

Proof: Put $a = 1$ in equation (6), we get the desired result.

Corollary 4.4. The second multiplicative plus Dakshayani index of HC_n is given by

$$DK_2^+ II(HC_n) = 2^{6n^2+6n} \times 3^{12n^2-6n} \times (2n^2-1)^{6n} \times (3n^2-2)^{6n^2-6n}.$$

Proof: Put $a = 2$ in equation (6), we obtain the desired result.

4. SILICATE NETWORKS

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A silicate network of dimension two is shown in Figure 3.

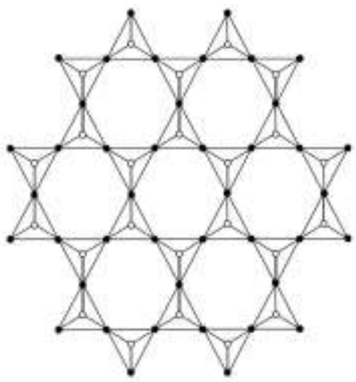

Figure 3. A 2- dimensional silicate network

Let G be the graph of a silicate network SL_n. By calculation, we obtained that G has $15n^2+3n$ vertices. In G, there are two types of vertices based on the degree of vertices as given in Table 3.

Table 3. Vertex partition of G and $\bar{G}$

$d_G(u) \setminus u \in V(G)$	3	6
$d_{\bar{G}}(u) \setminus u \in V(\bar{G})$	$15n^2 + 3n - 4$	$15n^2 + 3n - 7$
Number of vertices	$6n^2 + 6n$	$9n^2 - 3n$

In the following theorem, we compute the general Dakshayani index of SL_n.

Theorem 5. The general Dakshayani index of a silicate network SL_n is given by

$$DK^a(SL_n) = 3^a(15n^2 + 3n - 4)(6n^2 + 6n) + 6^a(15n^2 + 3n - 7)(9n^2 - 3n). \quad (7)$$

Proof: Let G be the graph of a silicate network SL_n. By using equation (1) and Table 3, we derive

$$DK^a(SL_n) = \sum_{u \in V(G)} d_{\bar{G}}(u) d_G(u)^a$$
$$= 3^a(15n^2 + 3n - 4)(6n^2 + 6n) + 6^a(15n^2 + 3n - 7)(9n^2 - 3n).$$

We obtain the following results by using Theorem 5.

Corollary 5.1. The first Dakshayani index of SL_n is given by

$$DK_1(SL_n) = \frac{105}{2}n^4 + 33n^3 - 14n^2 - \frac{9}{2}n.$$

Proof: Put $a = -1$ in equation (7), we get the desired result.

Corollary 5.2. The second Dakshayani index of SL_n is given by

$$DK_2(SL_n) = \frac{55}{4}n^4 + \frac{23}{2}n^3 - \frac{8}{3}n^2 - \frac{25}{12}n.$$

Proof: Put $a = -2$ in equation (7), we get the desired result.

In the following theorem, we compute the general multiplicative Dakshayani index of SL_n.

Theorem 6. The general multiplicative Dakshayani index of a silicate network SL_n is given by

$$DK^aII(SL_n) = 3^{a(6n^2+6n)} \times 6^{a(9n^2-3n)} \times (15n^2 + 3n - 4)^{6n^2+6n} \times (15n^2 + 3n - 7)^{9n^2-3n}. \quad (8)$$

Proof: Let G be the graph of a silicate network SL_n. By using equation (2) and Table 3, we deduce

$$DK^aII(SL_n) = \prod_{u \in V(G)} d_{\bar{G}}(u) d_G(u)^a$$
$$= \left[3^a(15n^2 + 3n - 4)\right]^{6n^2+6n} \times \left[6^a(15n^2 + 3n - 7)\right]^{9n^2-3n}.$$
$$= 3^{a(6n^2+6n)} \times 6^{a(9n^2-3n)} \times (15n^2 + 3n - 4)^{6n^2+6n} \times (15n^2 + 3n - 7)^{9n^2-3n}.$$

We obtain the following results by using Theorem 6.

Corollary 6.1. The first multiplicative Dakshayani index of SL_n is given by

$$DK_1II(SL_n) = 3^{-(6n^2+6n)} \times 6^{-(9n^2-3n)} \times (15n^2 + 3n - 4)^{6n^2+6n} \times (15n^2 + 3n - 7)^{9n^2-3n}.$$

Proof: Put $a = -1$ in equation (8), we get the desired result.

Corollary 6.2. The second multiplicative Dakshayani index of SL_n is given by

$$DK_2II(SL_n) = 3^{-(12n^2+12n)} \times 6^{-(18n^2-6n)} \times (15n^2 + 3n - 4)^{6n^2+6n} \times (15n^2 + 3n - 7)^{9n^2-3n}.$$

Proof: Put $a = -2$ in equation (8), we get the desired result.

Corollary 6.3. The first multiplicative plus Dakshayani index of SL_n is given by

$$DK_1^+II(SL_n) = 3^{6n^2+6n} \times 6^{9n^2-3n} \times (15n^2 + 3n - 4)^{6n^2+6n} \times (15n^2 + 3n - 7)^{9n^2-3n}.$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Proof: Put $a = 1$ in equation (8), we get the desired result.

Corollary 6.4. The second multiplicative plus Dakshayani index of SL_n is given by

$$DK_2^+ II(SL_n) = 3^{12n^2+12n} \times 6^{18n^2-3n} \times (15n^2+3n-4)^{6n^2+6n} \times (15n^2+3n-7)^{9n^2-3n}.$$

Proof: Put $a = 2$ in equation (8), we obtain the desired result.

5. CHAIN SILICATE NETWORKS

We consider a family of chain silicate networks. This network is denoted by CS_n and is obtained by arranging $n \geq 2$ tetrahedral linearly, see Figure 4.

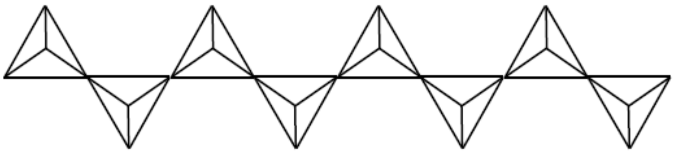

Figure 4. Chain silicate network

Let G be the graph of chain silicate network CS_n with $3n+1$ vertices. In G, there are two types of vertices based on the degree of vertices as given in Table 4.

Table 4. Vertex partition of G and $\overline{G}$

$d_G(u) \setminus u \in V(G)$	3	6
$d_{\overline{G}}(u) \setminus u \in V(\overline{G})$	$3n - 3$	$3n - 6$
Number of vertices	$2n + 2$	$n - 1$

In the following theorem, we compute the general Dakshayani index of CS_n.

Theorem 7. The general Dakshayani index of a chain silicate network CS_n is given by

$$DK^a(CS_n) = 3^a(3n-3)(2n+2) + 6^a(3n-6)(n-1). \quad (9)$$

Proof: Let G be the graph of a chain silicate network CS_n. By using equation (1) and Table 4, we deduce

$$DK^a(CS_n) = \sum_{u \in V(G)} d_{\overline{G}}(u) d_G(u)^a$$

$$= 3^a(3n-3)(2n+2) + 6^a(3n-6)(n-1).$$

We obtain the following results by using Theorem 7.

Corollary 7.1. The first Dakshayani index of CS_n is given by

$$DK_1(CS_n) = \frac{5}{2}n^2 - \frac{3}{2}n - 1.$$

Proof: Put $a = -1$ in equation (9), we get the desired result.

Corollary 7.2. The second Dakshayani index of CS_n is given by

$$DK_2(CS_n) = \frac{3}{4}n^2 - \frac{1}{4}n - \frac{1}{2}.$$

Proof: Put $a = -2$ in equation (9), we get the desired result.

In the following theorem, we compute the general multiplicative Dakshayani index of CS_n.

Theorem 8. The general multiplicative Dakshayani index of a chain silicate network CS_n is given by

$$DK^a II(CS_n) = 3^{a(2n+2)} \times 6^{a(n-1)} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}. \quad (10)$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Proof: Let G be the graph of a chain silicate network CS_n. By using equation (2) and Table 4, we deduce

$$DK^{a}II(CS_n) = \prod_{u \in V(G)} d_{\bar{G}}(u) d_G(u)^{a}$$
$$= \left[3^{a}(3n-3)\right]^{2n+2} \times \left[6^{a}(3n-6)\right]^{n-1}.$$
$$= 3^{a(2n+2)} \times 6^{a(n-1)} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}.$$

We obtain the following results by using Theorem 8.

Corollary 8.1. The first multiplicative Dakshayani index of CS_n is given by

$$DK_1II(CS_n) = \left(\frac{1}{3}\right)^{2n+2} \times \left(\frac{1}{6}\right)^{n-1} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}.$$

Proof: Put $a = -1$ in equation (10), we get the desired result.

Corollary 8.2. The second multiplicative Dakshayani index of CS_n is given by

$$DK_2II(CS_n) = \left(\frac{1}{9}\right)^{2n+2} \times \left(\frac{1}{36}\right)^{n-1} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}.$$

Proof: Put $a = -2$ in equation (10), we get the desired result.

Corollary 8.3. The first multiplicative plus Dakshayani index of CS_n is given by

$$DK_1^{+}II(CS_n) = 3^{2n+2} \times 6^{n-1} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}.$$

Proof: Put $a = 1$ in equation (10), we get the desired result.

Corollary 8.4. The second multiplicative plus Dakshayani index of CS_n is given by

$$DK_2^{+}II(CS_n) = 3^{4n+4} \times 6^{2n-2} \times (3n-3)^{2n+2} \times (3n-6)^{n-1}.$$

Proof: Put $a = 2$ in equation (10), we obtain the desired result.

REFERENCES

[1] V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
[2] H. Narumi and G. Katayama, Simple topological index, a newly devised index characterising the topological nuture structural isomers of saturated hydrocarbons, *Mem. Fac. Engin. Hokkido Univ.* 16 (1984) 209-214.
[3] I. Gutman and M. Ghorbani, Some properties of the Narumi-Katayama index, *Applied Math. Lett.* in press.
[4] D.J. Klein and V.R. Rosenfeld, The degree product index of Narumi and Katayama, *MATCH Commun. Math. Comput. Chem.* 64(2010) 607-618.
[5] Ž. Tomović and I. Gutman, Namuri-Katayama index of phenylenes. *J. Serb. Chem. Soc.*, 66 (2001) 243-247.
[6] R. Todeshini and V. Consonni, New local vertex invariants and descriptors based on functions of vertex degrees, *MATCH Commun. Math. Comput. Chem.* 64(2010) 359-372.
[7] R. Todeschini, D. Ballabio and V. Consonni, *Noval molecular descriptors based on the functions of new vertex degrees* in : I Gutman, B. Furtula (Eds). *Novel molecular structure Descriptors – Theory and Applications I*, Univ. Kragujevac, Kragujevac, 2010, 72-100.
[8] M. Eliasi, A. Irammanesh and I. Gutman, Multiplicative versions of first Zagreb index, *MATCH Commun. Math. Comput. Chem.* 68(2012) 217-230.
[9] I. Gutman, Multiplicative Zagreb indices of trees, *Bull, Internat. Math. Virt. Inst.* 1(2011) 13-19.
[10] V.R.Kulli, Dakshayani indices, submitted.
[11] S. Wang, J.B. Liu, C. Wang and S. Hayat, Further results on computation of topological indices of certain networks, *arXiv:* 1605.00253v2 [math. CO] 5 May 2016, 1-6.

CITE AN ARTICLE

Kulli, V. R. (2018). MULTIPLICATIVE DAKSHAYANI INDICES. *INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY*, *7*(10), 75-82.